Cellular automata model simulating traffic interactions between on-ramp and main road

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In this paper, we study the on-ramp system using the cellular automata traffic flow model. Different from previous works, we consider not only the influence of the on-ramp flow on the main road but also the opposite influence. The update rules are given in detail and the concept of priority is introduced. The numerical simulations are carried out and the phase diagram is presented. Two different types of phase diagram are distinguished and the currents of the on-ramp system are discussed.

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I. INTRODUCTION

In the past few decades, traffic problems have attracted the interest of a community of physicists [1–3]. Traffic flow, a kind of many-body systems of strongly interacting vehicles, shows various complex behaviors. Numerous empirical data of the highway traffic have been obtained, which demonstrate the existence of qualitatively distinct dynamic states [4–7]. In particular, three distinct dynamic phases are observed on highways: the free traffic flow, the traffic jam, and the synchronized traffic flow. It has been found out experimentally that the complexity in traffic flow is linked to diverse space-time transitions between the three basically different kinds of traffic [4].

To understand the behavior of traffic flow, various traffic flow models have been proposed and studied, including car-following models, cellular automaton (CA) models, gas-kinetic models, and hydrodynamic models [8–16]. With the help of these models, free flow and jams are well understood. On the other hand, the nature of synchronized traffic flow remains unclear despite various efforts [4,5,16,17].

Recent experimental investigation shows that in the majority of cases, synchronized traffic is observed localized near bottlenecks and thus it is believed that bottlenecks are important for the formation of synchronized traffic. The bottlenecks include on-ramps, off-ramps, lane closings, uphill gradients, narrow road sections, etc. Among the various types of bottlenecks, the on-ramp is of particular interest to researchers and has been widely studied [16–21]. Popkov *et al.* [21] compared the experimental data from an on-ramp with simulations of a cellular automata model. It is concluded that the dynamics due to the ramp can be described by a bottleneck. However, the traffic on the ramp is not modeled explicitly in their work.

Lee *et al.* [16] and Helbing *et al.* [17] simulated the on-ramp using the macroscopic models. They incorporated the influence of the on-ramp into the models by adding a source term to the continuity equation. It was found that different types of congested traffic states may occur according to different inflows on the main road and the on-ramp. So it was proposed that the synchronized traffic may not be a single dynamic phase but rather a collection of multiple phases.

Diedrich *et al.* [18] and Campari and Levi [19] simulated the on-ramp using the CA models. The on-ramp is incorporated into the models as follows. Within each time step, the lattice will be searched in the region of the on-ramp either successively or stochastically until a vacant cell is found. Then a car will be inserted into this cell with a chosen probability. Traffic phenomena such as synchronized flow, the lane inversion, and phase separation were reproduced.

The on-ramp has also been simulated from car-following models [20]. In the simulations, vehicles enter the road from the on-ramp if a safety distance both to the car in front and that behind is given. The traffic states found turn out to be qualitatively the same as in macroscopic model simulation.

Nevertheless, in all the above simulations, only the influence of the on-ramp on the main road is considered, and the opposite influence is ignored. In fact, from the results in this paper, it is found that the main road flow also has an important effect on the on-ramp, e.g., the main road plays the role as a bottleneck to the on-ramp under some conditions.

In this paper, we study the interactions between the traffic flows on the main road and on-ramp. We use the CA model because compared with other dynamical approaches, CA models are conceptually simpler, and can be easily implemented on computers for numerical investigations.

The paper is organized as follows. In Sec. II, the traffic model for on-ramp simulation is presented. In Sec. III, the numerical results are analyzed. The conclusions are given in Sec. IV.

II. MODEL

Motions of cars and interactions between cars are the microscopic processes in the traffic flow. One of the approaches to microscopic traffic processes is based on cellular automata. A CA model treats the motion of cars as hopping processes on one-dimensional lattices. The Wolfram's rule-184 CA is the simplest choice [22]. Nagel and Schreckenberg introduced their CA model (NS model) by extending the 184 CA to consider the high velocity and the stochastic processes [9]. They showed that start-stop waves appear in the congested traffic region as observed in real freeway traffic. From then on, the NS model has been widely studied and many related models are proposed [10]. In this paper, we use the

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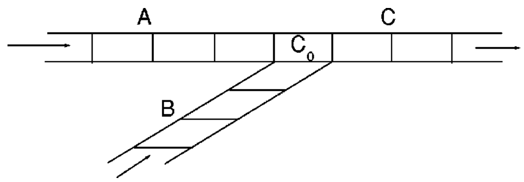

FIG. 1. Sketch of the on-ramp system.

NS model to serve as the basis model for the implementation of the on-ramp.

Before we start with our considerations concerning the on-ramp, let us recall the update rules (parallel dynamics) in the NS model: $R1$ acceleration, $v_i \rightarrow \min(v_{max};v_i+1)$; $R2$, braking, $v_i \rightarrow \min(d_i;v_i)$; $R3$, randomization, $v_i \rightarrow \max(0;v_i-1)$ with probability p; $R4$, driving, car i moves v_i cells.

Here v_i and d_i denote the velocity of car i and the number of empty cells in front of car i, respectively. The maximum velocity and the slowdown parameter are denoted as v_{max} and p, respectively. In this paper, we focus on the deterministic case $p=0$, i.e., step $R3$ is ignored.

Next we discuss the implementations of the on-ramp. To make the problem simple, we assume that the main road is single lane and the on-ramp connects the main road only on one lattice C_0 as shown in Fig. 1. To describe the problem conveniently, we denote the main road upstream of C_0, the on-ramp, and the main road downstream of C_0 (includes lattice C_0) as roads A, B, and C, respectively (see Fig. 1).

We denote the leading cars on roads A and B as A_{lead} and B_{lead}, the last car on road C as C_{last}. Thus, we can consider the update of the system.

For one time step, we first examine whether the car A_{lead} can arrive at lattice C_0 or not. If it cannot, then it is obvious that the update of the cars on road A is not affected by those cars on road B, and also the update of the cars on road B is not affected by those cars on road A in this time step. Similarly, if the car B_{lead} cannot arrive at lattice C_0 in one time step, the updates of cars on both roads A and B are not affected by each other in this time step.

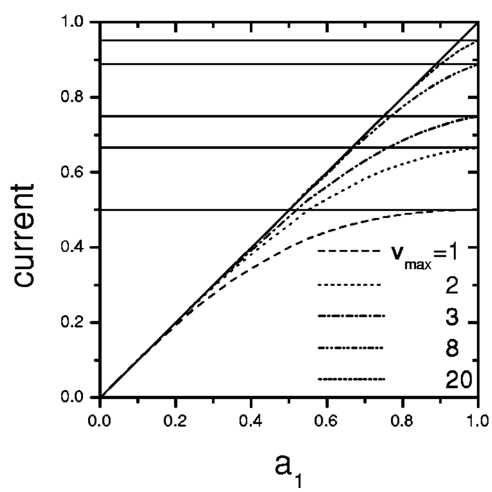

FIG. 2. The current on road A in the case of $a_2=0$. The solid line denotes the line with slope 1, the horizontal line represents the maximum flow $J_{max}=v_{max}/(1+v_{max})$.

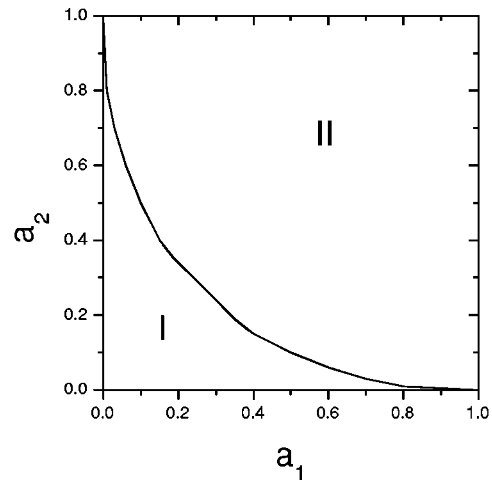

FIG. 3. Phase diagram of the on-ramp system in the case of $v_{max}=1$.

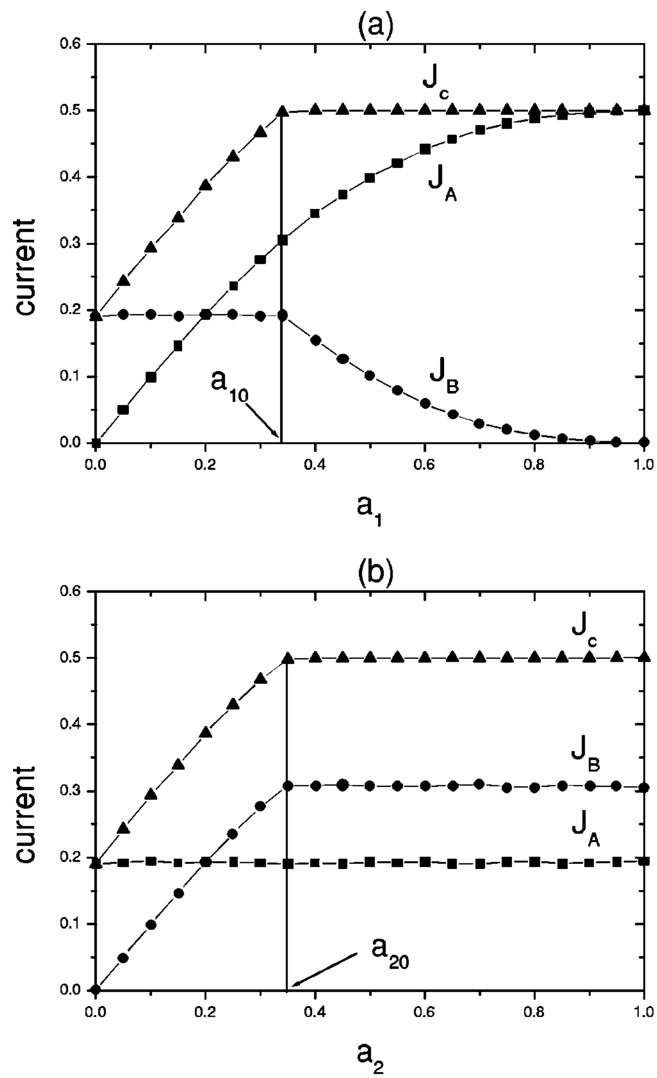

FIG. 4. Dependence of the currents J_A, J_B, and J_C on a_1 and a_2 in the case of $v_{max}=1$. (a) a_2 is fixed at 0.2, (b) a_1 is fixed at 0.2.

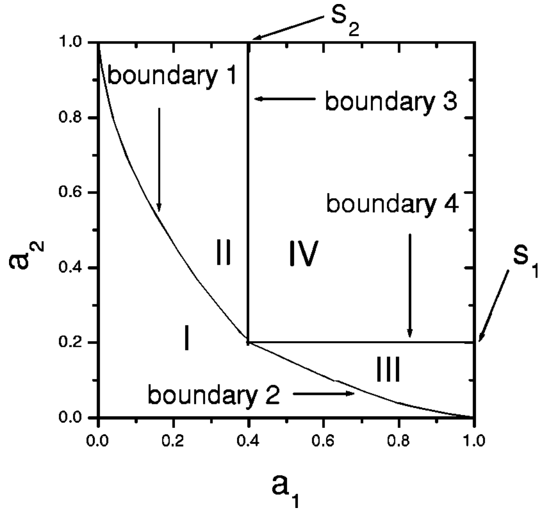

FIG. 5. Phase diagram of the on-ramp system in the case of $v_{max}=5$.

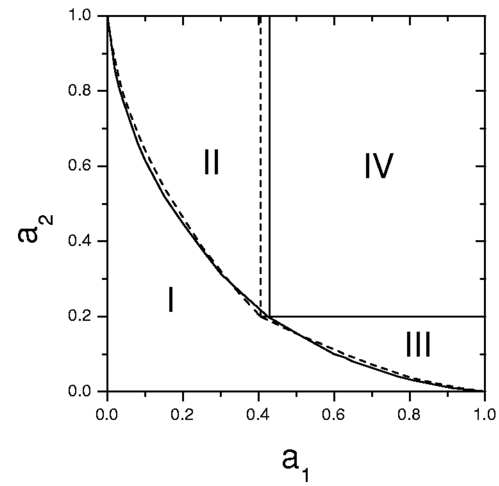

FIG. 6. Phase diagram of the on-ramp system in the case of $v_{max}=2$ (solid lines) and $v_{max}=3$ (dashed lines).

Thus, we come to the point that both cars A_{lead} and B_{lead} can arrive at C_0 in one time step. For this case, we can calculate the time t_a and t_b needed to arrive at C_0 for cars A_{lead} and B_{lead}:

$$t_a=\frac{x_{C_0}-x_{A_{lead}}}{\min(v_{max},x_{C_{last}}-x_{A_{lead}}-1,v_{A_{lead}}+1)}, \tag{1}$$

$$t_b=\frac{x_{C_0}-x_{B_{lead}}}{\min(v_{max},x_{C_{last}}-x_{B_{lead}}-1,v_{B_{lead}}+1)}, \tag{2}$$

where x denotes the position of the car or the lattice and v denotes the velocity of the car.

If $t_a<t_b$, we can suppose that car A_{lead} has the priority to occupy lattice C_0. For this case, the update of the cars on road A will not be affected by the cars on road B in this time step. However, the update of the cars on road B will be affected by cars on road A. To implement the update, two substeps are classified: (i) the update of cars on roads A and C as usual, and the leading car on road A becomes the new last car on road C; (ii) the update of cars on road B. For $t_a>t_b$, the situation is the similar, just the roles of roads A and B interchange.

As for $t_a=t_b$, it is reasonable to endow the car nearer to the lattice C_0 (provided the distances to C_0 are different for cars A_{lead} and B_{lead}) with priority, and the problem becomes the same as the case $t_a>t_b$ or $t_a<t_b$.

Finally, if $t_a=t_b$ and the distances to C_0 are the same for cars A_{lead} and B_{lead}, a reasonable solution is to endow the car A_{lead} with priority because it is on the main road. The situation turns to be the same as $t_a<t_b$. We will see later that this rule causes asymmetry in the phase diagram because all the rules except this one are symmetric with respect to roads A and B.

The boundary conditions are adopted as follows. We assume that the left most cells on roads A and B correspond to $x=1$, and the entrance sections of roads A and B include v_{max} cells, i.e., the cars can enter roads A and B from the cells $1,2,\ldots,v_{max}$. In one time step, when the update of the cars on the road is completed, we check the positions of the last cars on roads A and B and that of the leading car on road C, which are denoted as $x_{A_{last}}$, $x_{B_{last}}$, and $x_{C_{lead}}$, respectively. If $x_{A_{last}}(x_{B_{last}})>v_{max}$, a car with velocity v_{max} is injected with probability $a_1(a_2)$ at the cell $\min[x_{A_{last}}(x_{B_{last}})-v_{max},v_{max}]$. Near the exit of the road C, the leading car is removed if $x_{C_{lead}}>L_C$ (L_C denotes the position of the right most cell on road C) and the following car becomes the new leading car and it moves without any hindrance.

III. NUMERICAL RESULTS

As a preliminary work, we consider the special case that there are no cars on one road, say, road B. This means $a_2=0$, and the problem reduces to the deterministic NS model problem in open boundary conditions. The removal rate on the right boundary is 1 and the injection rate on the left boundary is a_1. The numerical simulation results are shown in Fig. 2. In the simulations of this paper, roads A, B, and C are divided into $100\times v_{max}$ cells, and the first 40 000 time steps are discarded to let the transient time die out. The current is obtained by counting the vehicles that pass a virtual detector in 100 000 time steps.

In the deterministic NS model, the maximum flow is $J_{max}=v_{max}/(1+v_{max})$, which is represented by the horizontal lines in Fig. 2. Thus, one can see that when a_1 is small, the curve of the current against a_1 is a straight line with the slope 1. Then with the increase of a_1, the curve begins to deviate from the straight line and it bends downward but still increases with a_1. When $a_1=1$, the current reaches the maximum flow. One also notes that with the increase of v_{max}, the deviation from the straight line occurs later.

Next we consider the case $v_{max}=1$. In Fig. 3, the phase diagram in the (a_1, a_2) space is shown and two regions are categorized. In region I, flows on both roads A and B are

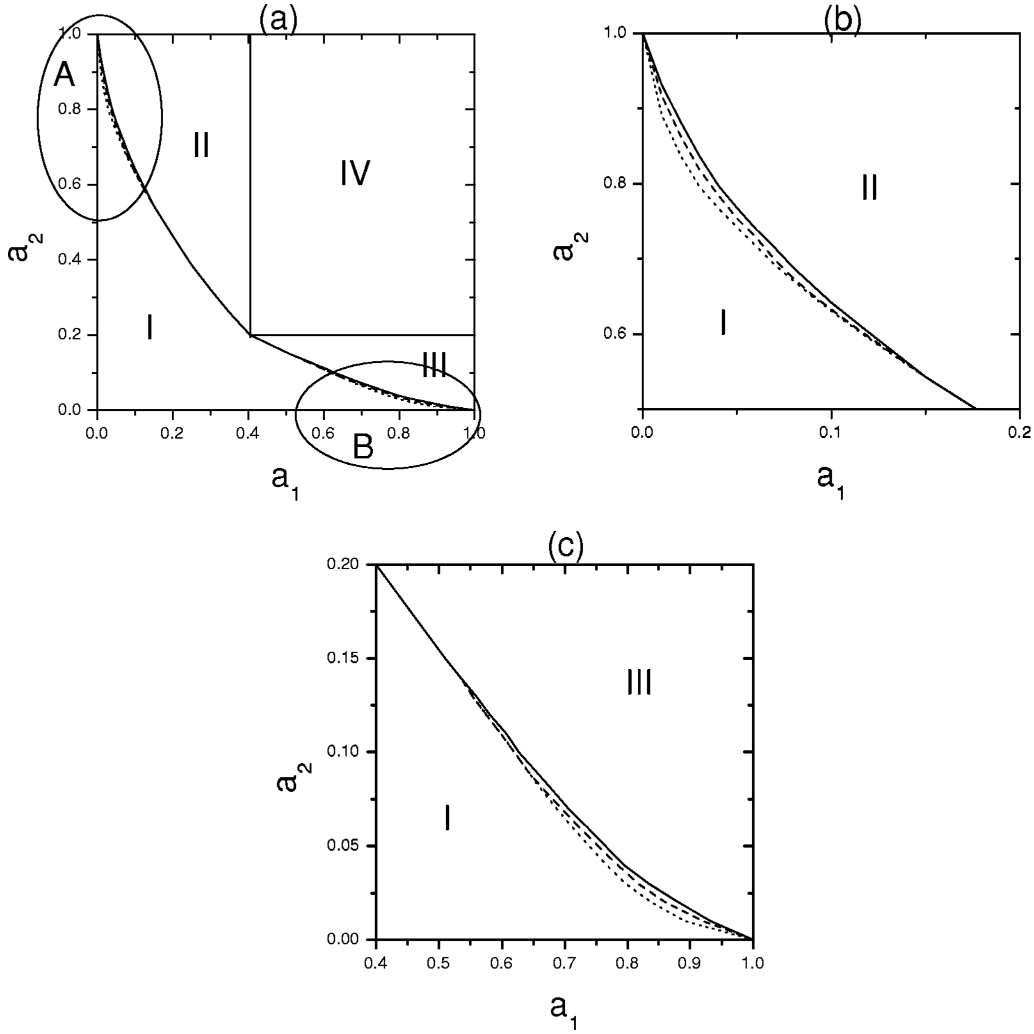

FIG. 7. (a) Phase diagrams of the on-ramp system in the case of $v_{max}=3$ (solid lines), $v_{max}=8$ (dashed lines), and $v_{max}=20$ (dotted lines). (b) Magnification of part *A*. (c) Magnification of part *B*.

free, and in region II, it is still free flow on road *A* but becomes congested on road *B*. In other words, the main road plays the role as a bottleneck to the on-ramp in region II.

We give the currents J_A, J_B, and J_C on roads *A*, *B*, and *C* in Fig. 4 (due to the conservation of the number of the vehicles, the currents are related through $J_A+J_B=J_C$). In Fig. 4(a), the typical curves are plotted at the fixed $a_2=0.2$. One finds that two ranges of a_1 can be classified.

(1) When $a_1<a_{10}$, the flows on both roads *A* and *B* are free and the traffic pattern is in region I. J_B remains constant and J_A increases with a_1. As a result, J_C increases with a_1.

(2) When $a_1>a_{10}$, the flow on road *B* becomes congested and it is still free on road *A*. One finds that J_B decreases with the increase of a_1 and J_A still increases with a_1. Being the sum of J_A and J_B, J_C turns into the constant 0.5.

In Fig. 4(b), the typical curves are plotted at the fixed $a_1=0.2$. Here, a_2 can also be classified into two ranges.

(1) When $a_2<a_{20}$, the flows on both roads *A* and *B* are free. J_A remains constant and J_B increases with a_2. As a result, J_C increases with a_2.

(2) When $a_2>a_{20}$, the flow on road *B* becomes congested and it is still free on road *A*. Thus, J_B becomes a constant and J_A still remains constant. As a result, J_C again turns into the constant 0.5.

Now we study the traffic situations arising in the case of $v_{max}\geqslant 2$. The simulations show that the phase diagram for the case is quite different from that of $v_{max}=1$ (see Figs. 5–7). The phase diagram is classified into four regions. In region I, the traffic flows on both roads *A* and *B* are free flow; in region II, the traffic is congested on road *B* and it is free flow on road *A*; in region III, the traffic is congested on road *A* and is free on road *B*; in region IV, the traffic flows are congested on both roads. This means that in region II (III), the main road (on-ramp) plays the role of a bottleneck for the on-ramp (main road).

We investigate the influence of v_{max} on the phase diagram. In Fig. 6, the phase diagrams of $v_{max}=2$ and $v_{max}=3$ are shown. One can see that from $v_{max}=2$ to $v_{max}=3$, region IV slightly expands and the boundaries 1 and 2 (the four boundaries between the four regions are denoted as boundaries 1–4 as shown in Fig. 5) slightly change. When $v_{max}\geqslant 3$, the simulations show that region IV will remain invariant and only a very tiny distortion of boundaries 1 and 2 exists (see Fig. 7). Summarizing, the phase diagram in the

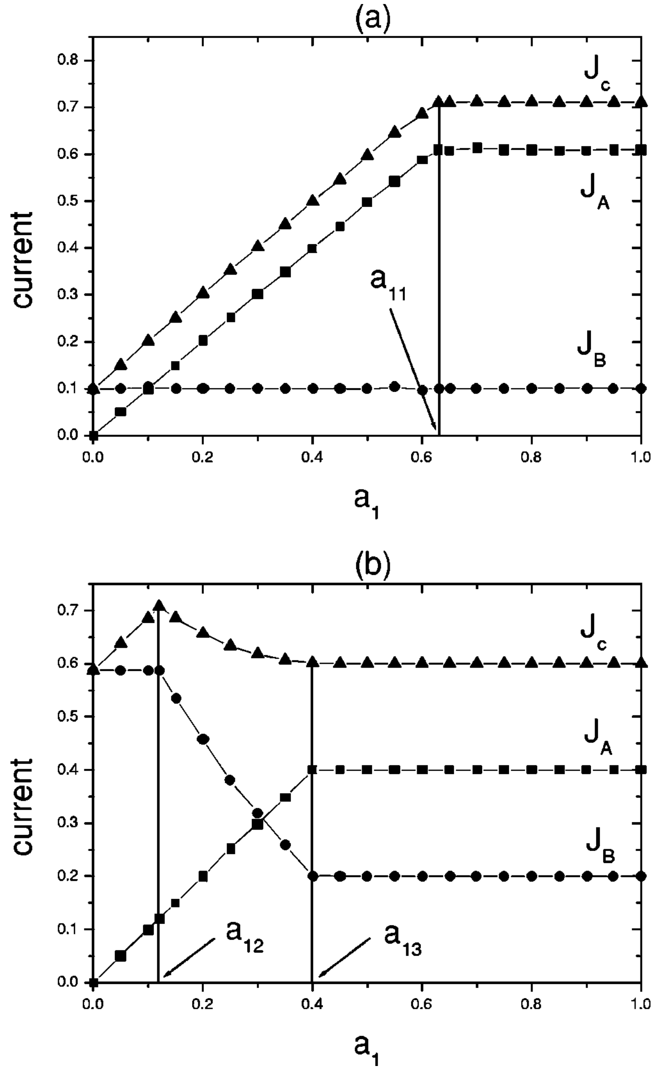

FIG. 8. Dependence of the currents J_A, J_B, and J_C on a_1 with a_2 fixed in the case of $v_{max}=5$. (a) $a_2=0.1$, (b) $a_2=0.6$.

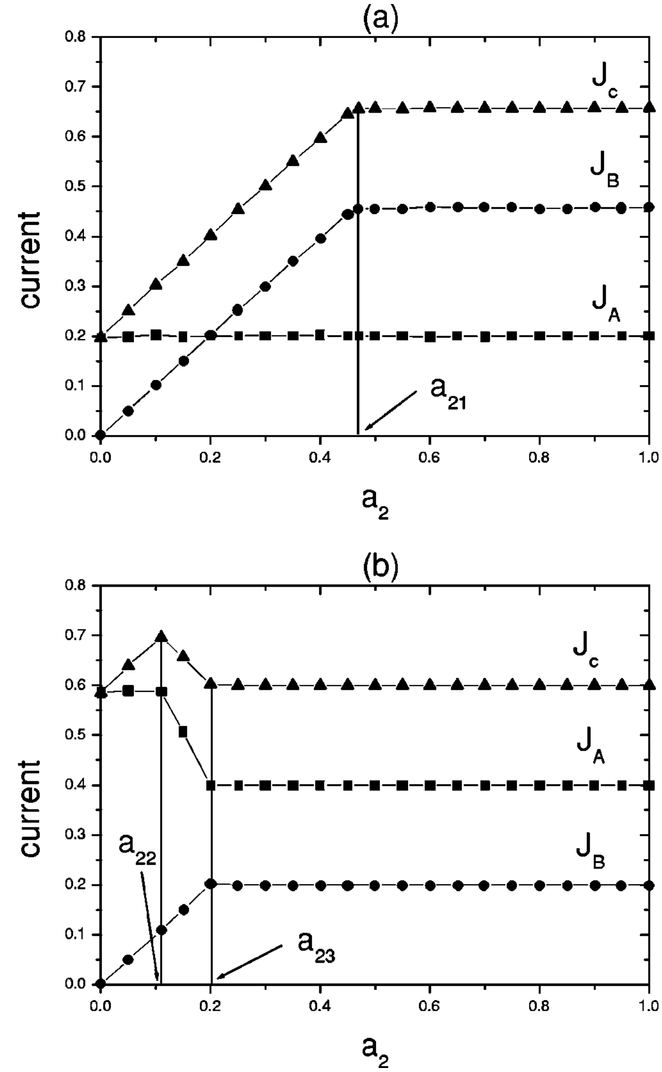

FIG. 9. Dependence of the currents J_A, J_B, and J_C on a_2 with a_1 fixed in the case of $v_{max}=5$. (a) $a_1=0.2$, (b) $a_1=0.6$.

case of $v_{max}\geqslant 2$ depends very weakly on v_{max}.

As in the case of $v_{max}=1$, we focus on the currents J_A, J_B and J_C. Here we take $v_{max}=5$, for example. The vertical coordinate of boundary 4 and the horizontal coordinate of boundary 3 are calculated to be $s_1=0.2$ and $s_2=0.4$ (see Fig. 5).

First, we fix the value of a_2 and study the dependence of the three currents on a_1. In Fig. 8, the typical curves are given. Here, two representative values of a_2 are chosen: the first is smaller than s_1 and the second is greater than s_1.

In Fig. 8(a), $a_2=0.1$ is smaller than s_1. In this case, a_1 is classified into two ranges.

(1) When $a_1<a_{11}$, the flows on both roads A and B are free and the traffic pattern is in region I. J_B remains constant and J_A increases with a_1. As a result, J_C increases with a_1.

(2) When $a_1>a_{11}$, the flow on road A becomes congested and it is still free on road B. Thus, J_A becomes a constant and J_B still remains constant. As a result, J_C turns into the constant.

In Fig. 8(b), $a_2=0.6$ is greater than s_1. In this case, a_1 is classified into three ranges.

(1) When $a_1<a_{12}$, the flows on both roads A and B are free and the traffic pattern is in region I. J_B remains constant and J_A increases with a_1. As a result, J_C increases with a_1.

(2) When $a_{12}<a_1<a_{13}$, the flow on road B becomes congested and it is still free on road A. One finds that J_B decreases with the increase of a_1 and J_A still increases with a_1. Note that being the sum of J_A and J_B, J_C decreases with the increase of a_1.

(3) When $a_1>a_{13}$, the flows on both roads are congested. For this case, J_A becomes constant and J_B changes into another constant that is smaller than the initial one. As a result, J_C again turns into the constant.

Second, we fix the value of a_1 and study the dependence of the three currents on a_2. In Fig. 9, the typical curves are given. Here, we still choose two representative values of a_1: in Fig. 9(a), $a_1=0.2$ is smaller than s_2; in Fig. 9(b), $a_1=0.6$ is greater than s_2. It can be seen from Figs.9(a) and 9(b) that the similar results can be obtained as in the case of fixed a_2 with just the roles of roads A and B interchanged.

Finally, we explain why the phase diagram for the case

$v_{max}=1$ is different from that in the case $v_{max}\geqslant 2$. For the purpose, we focus on the current J_C. The simulations show that for $v_{max}\geqslant 2$, $J_C=0.6$ in region IV and $0.6<J_C<J_{max}$ in regions II and III. This implies that the existence of such a kind of phase diagram that consists of four phases requires that $J_{max}>0.6$. As for $v_{max}=1$, the maximum flow $J_{max}=0.5<0.6$. Therefore, the phase diagram consisting of four phases does not exist, and the phase diagram composed of two phases occurs instead.

IV. CONCLUSIONS

In traffic flow researches, the on-ramp is of particular interest. Nevertheless, the previous works only concentrated on the influence of the on-ramp flow on the main road, and seldom considered the opposite influence. For this reason, in this paper, we study the traffic interactions between on-ramp and main road using the cellular automata traffic flow model.

To make the problem simple, we only consider the merging of a single-lane road with the on-ramp. The update rules are given in details and the concept of priority is introduced. The numerical simulations show that two different types of phase diagrams exist: one under the case $v_{max}=1$ and the other under $v_{max}>1$. For $v_{max}=1$, two regions are classified but for $v_{max}>1$, four regions are recognized. The dependence of the currents on roads *A*, *B*, and *C* on the values of a_1 and a_2 has also been investigated.

In the present paper we have confined ourselves to the deterministic rule. It will be more interesting to study the cases where the randomization is considered, and in fact the study is now under progress.

ACKNOWLEDGMENTS

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[1] *Traffic and Granular Flow '97*, edited by M. Schreckenberg and D. E. Wolf (Springer, Singapore, 1998); *Traffic and Granular Flow '99*, edited by D. Helbing, H.J. Herrmann, M. Schreckenberg, and D. E. Wolf (Springer, Berlin, 2000).
[2] D. Chowdhury, L. Santen, and A. Schadschneider, Phys. Rep. **329**, 199 (2000).
[3] D. Helbing, Rev. Mod. Phys. **73**, 1067 (2001).
[4] B.S. Kerner and H. Rehborn, Phys. Rev. E **53**, R1297 (1996); **53**, R4275 (1996); Phys. Rev. Lett. **79**, 4043 (1997); B.S. Kerner, *ibid.* **81**, 3797 (1998); J. Phys. A **33**, L221 (2000).
[5] H.Y. Lee, H.W. Lee, and D. Kim, Phys. Rev. E **62**, 4737 (2000).
[6] D. Helbing, Phys. Rev. E **55**, R25 (1997); **55**, 3735 (1997); M. Treiber and D. Helbing, J. Phys. A **32**, L17 (1999).
[7] M. Treiber, A. Hennecke, and D. Helbing, Phys. Rev. E **62**, 1805 (2000).
[8] M. Bando *et al.*, Phys. Rev. E **51**, 1035 (1995); D. Helbing and B. Tilch, *ibid.* **58**, 133 (1998); R. Jiang, Q.S. Wu, and Z.J. Zhu, *ibid.* **64**, 017101 (2001); E. Tomer, L. Safonov, and S. Havlin, Phys. Rev. Lett. **84**, 382 (2000).
[9] K. Nagel and M. Schreckenberg, J. Phys. I **2**, 2221 (1992).
[10] S.C. Benjamin, N.F. Johnson, and P.M. Hui, J. Phys. A **29**, 3119 (1996); R. Barlovic *et al.*, Eur. Phys. J. B **5**, 793 (1998); W. Knospe *et al.*, J. Phys. A **33**, L477 (2000); X.B. Li, Q.S. Wu, and R. Jiang, Phys. Rev. E **64**, 066128 (2001).
[11] D. Helbing, A. Hennecke, V. Shvetsov *et al.*, Transp. Res., Part B: Methodol. **35B**, 183 (2001).
[12] M.J. Lighthill and G.B. Whitham, Proc. R. Soc. London, Ser. A **229**, 317 (1955).
[13] H.J. Payne, in *Mathematical Models of Public Systems*, edited by G.A. Bekey (Simulation Council, La Jolla, 1971), Vol. 1.
[14] A. AW and M. Rascle, SIAM (Soc. Ind. Appl. Math.) J. Appl. Math. **60**, 916 (2000).
[15] R. Jiang, Q.S. Wu, and Z.J. Zhu, Chin. Sci. Bull. **46**, 345 (2001); Transp. Res., Part B: Methodol. **36**, 405 (2002).
[16] H.Y. Lee, H.W. Lee, and D. Kim, Phys. Rev. Lett. **81**, 1130 (1998); Phys. Rev. E **59**, 5101 (1999).
[17] D. Helbing and M. Treiber, Phys. Rev. Lett. **81**, 3042 (1998); D. Helbing, A. Hennecke, and M. Treiber, *ibid.* **82**, 4360 (1999).
[18] G. Diedrich *et al.*, Int. J. Mod. Phys. C **11**, 335 (2000).
[19] E.G. Campari and G. Levi, Eur. Phys. J. B **17**, 159 (2000).
[20] P. Berg and A. Woods, Phys. Rev. E **64**, 035602 (2001).
[21] V. Popkov *et al.*, J. Phys. A **34**, L45 (2001).
[22] S. Wolfram, Rev. Mod. Phys. **55**, 601 (1983).